Received: *November 30, 2000,* Accepted: *December 06, 2000*

A note on dilaton absorption and near-infrared D3-brane holography

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Abstract: We consider the first subleading terms in the low-energy cross section for the absorption of dilaton partial waves by D3-branes. We demonstrate that these corrections, computed previously via supergravity, can be reproduced exactly in a worldvolume calculation using a deformation of $\mathcal{N} = 4$ SYM theory by a dimension eight chiral operator. The calculation does not depend on how the theory is regularized. This result provides another hint that holographic duality between the D3-brane worldvolume theory and the corresponding supergravity solution may be valid beyond the near horizon limit.

Keywords: AdS/CFT Correspondance, 1/N Expansion, p-branes.

JHEP12(2000)005

Contents

1. Introduction

The study of particle absorption by D-branes [1, 2, 3] provided one of the early hints of an exact correspondence between the gauge theories living on branes and gravitational physics in the corresponding supergravity p-brane backgrounds. The correspondence is suggested by the existence of two different pictures of the absorption process.

In the first, the absorption is viewed semiclassically as a wave propagating in the appropriate p-brane supergravity solution and being "absorbed" at the horizon. The cross section is determined by solving the wave equation for the particle of interest in this geometry with the boundary condition that the wave is purely ingoing at the horizon.

The second ("worldvolume") picture treats the incident particle as an excitation in the field theory on the brane which is absorbed by decaying quantum mechanically into two or more particles confined to the brane world volume. From this point of view, the cross section may be most easily determined by computing the two point function of the worldvolume operator which couples to the bulk supergravity particle of interest.

Probably the simplest example to consider is the absorption of a minimally coupled scalar by a stack of N D3-branes [1, 2, 4]. The supergravity description is reliable if the energy ω of the incident wave is small and the curvature radius R is large in string units, $\omega\sqrt{\alpha'} \ll 1$, $R/\sqrt{\alpha'} \gg 1$. As already noticed in [1], the dimensionless combination ωR can be kept fixed and arbitrary within the supergravity regime. In the low-energy limit $\omega R \ll 1$ the interaction of the incoming partial wave effectively takes place in the near-horizon AdS region. In the field theory picture, this is the limit in which the brane degrees of freedom decouple from the bulk and are controlled by the superconformal $\mathcal{N}=4$ SYM theory: this is part of the motivation for the standard AdS/CFT correspondence [5] between the $\mathcal{N}=4$ theory and supergravity/string theory on $AdS_5 \times S^5$.

Exact agreement has been demonstrated between the supergravity and worldvolume calculations of the leading low-energy absorption cross sections for all partial waves of the dilaton field [6]. For the l-th partial wave, the result of both calculations is [3, 6]

$$\sigma^l_{wv} = \sigma^l_{sg} = \frac{\pi^4}{24} \frac{(l+3)(l+1)}{[(l+1)!]^4 2^{4l}} R^{4l+8} \omega^{4l+3} , \tag{1.1}$$

where $R^4 = 4\pi g N \alpha'^2$. Since the field theory computation is performed in the free field theory approximation, while supergravity is valid at strong 't Hooft coupling gN, this exact agreement can only be explained by assuming a non-renormalization theorem [2] for the 2-point functions of the $\mathcal{N}=4$ SYM operators dual to the dilaton partial waves.[1]

Given the precise agreement between the leading order cross sections in the two pictures, it is interesting to ask what happens away from the low energy limit. On the supergravity side, the absorption cross section has been determined exactly by Gubser and Hashimoto [11] as a perturbative expansion to all orders in ωR with the result

$$\begin{aligned} \sigma^l &= \sigma^l_0 \left(1 + \sum_{n=1}^{\infty} \sum_{k=0}^{n} b^l_{n,k} (\omega R)^{4n} (\log(\omega \bar{R}))^k \right) \\ &= \sigma^l_0 \left(1 + b^l_{1,1} (\omega R)^4 \log(\omega \bar{R}) + b^l_{1,0} (\omega R)^4 + \cdots \right) , \end{aligned} \tag{1.2}$$

where $\bar{R} = \gamma R/2$ and $b^l_{n,k}$ is a series of numerical coefficients given implicitly in [11].[2] In this paper, we ask whether it is possible to reproduce any of the correction terms (i.e. determine the coefficients $b^l_{n,k}$) through a dual calculation in the worldvolume theory.

Away from the low-energy limit, the worldvolume theory of the branes is no longer described simply by $\mathcal{N}=4$ SYM theory. At weak string coupling, the corrections (for a slowly varying field strength) are given by the incompletely known non-abelian Born-Infeld action,

$$S = \frac{1}{2\pi g} \int d^4x \left\{ -\frac{1}{4} \operatorname{Tr}(F^2) + \frac{1}{8} (2\pi\alpha')^2 \operatorname{STr} \left(F^4 - \frac{1}{4} (F^2)^2 \right) + \cdots \right\} , \tag{1.3}$$

[1] Non-renormalization theorems for the $\mathcal{N}=4$ theory are now widely believe to hold for all two and three point functions of chiral operators [7, 26], and even for some special ("extremal" and "next-to-extremal") n-point functions [8, 9]. See also [10].

[2] For generalizations of this result see e.g. [12].

where we have written only the gauge field terms. It is important to note that the dimension eight term in the lagrangian,

$$\mathcal{O}_8^{\mathrm{BI}} = \frac{\pi}{4g}\,\mathrm{STr}\left(F^4 - \frac{1}{4}(F^2)^2 + \cdots\right) \tag{1.4}$$

lies in a short multiplet of the $\mathcal{N} = 4$ supersymmetry algebra,[3] so that its dimension is protected for any value of the coupling. On the other hand, the higher order corrections in (1.3) are operators in long multiplets (dual to string states) and acquire large anomalous dimensions in the limit of strong 't Hooft coupling $gN \to \infty$. Based on these observations, Gubser and Hashimoto suggested tentatively that the lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{SYM}} + c\,(\alpha')^2 \mathcal{O}_8^{\mathrm{BI}} \tag{1.5}$$

might be used by itself at large gN to reproduce all correction coefficients $b^l_{n,k}$ in the full absorption cross section. Here, the coefficient c is included since the normalization of $\mathcal{O}_8$ in the strong coupling lagrangian is not automatically the same as in the Born Infeld action. An even stronger claim in this direction has been made by Intriligator, who argued that a four-dimensional field theory with lagrangian (1.5) (for a particular value of the coefficient c) is holographically dual to the full type-IIB *string* theory on the D3-brane geometry, for arbitrary values of g and N.

Claims of this nature seem problematic for a variety of reasons. Firstly, it is not clear what the theory (1.5) means, since the operator $\mathcal{O}_8$ is not renormalizable. One possible definition would be as a Wilsonian action with a physical cutoff at some scale of order R; however, one must somehow introduce a cutoff in the theory while preserving all the supersymmetry. In [14], Intriligator takes the point of view that for each value of c, there is a unique theory with 16 supercharges whose lagrangian is (1.5) in the near infrared, and argues that the form of the lagrangian for these theories is precisely (1.5) along the entire RG flow (though it is unclear what the UV fixed point could be).

Even if there is a sensible way to define a field theory based on the lagrangian (1.5), an exact duality with supergravity/string theory on the D3-brane geometry would be surprising, since without taking the usual near horizon limit, the degrees of freedom on the brane do not decouple from those of the bulk. From this point of view, it appears that any theory dual to supergravity on the full D3-brane geometry should include both brane and bulk degrees of freedom, as emphasized in [15].[4]

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[3] In fact, it is the only single-trace non-renormalizable operator which is in a short multiplet, is a Lorentz scalar and preserves the SU(4) R-symmetry. The same operator arises in the low-energy effective action of $\mathcal{N} = 4$ SYM on the Coulomb branch, where it leads to logarithmic corrections to absorption by split D3-branes. The leading correction to dilaton s-wave absorption by a double-centered D3-brane geometry has been matched exactly with field theory in [13].

[4] See also [16] for another approach to D3-brane holography.

In this note, our goals will be more modest than trying to determine a holographic dual to the full D3-brane geometry. We simply assume that in the near infrared, and at strong 't Hooft coupling, the worldvolume degrees of freedom on the brane are governed by a theory of the form (1.5). We then compute the two point function of the dilaton operator in this deformation of $\mathcal{N} = 4$ SYM theory in an attempt to reproduce the leading correction $b^l_{1,1}$. We will see that the required computation is independent of the regularization scheme so all field theory calculations are well defined. Our strategy will be to fix the coefficient of $\mathcal{O}_8$ in (1.5) by requiring that the leading correction for the s-wave cross section ($l = 0$) matches with the supergravity result. Having fixed this normalization, we compute the remaining coefficients $b^l_{1,1}$ for all higher partial waves. We find that the result,

$$b^l_{1,1} = -\frac{1}{(l+1)(l+2)(l+3)}\,, \tag{1.6}$$

is in precise agreement with the supergravity calculation of [11]. Thus, by choosing a single coefficient in the worldvolume theory (the normalization of $\mathcal{O}_8$), we are able to reproduce the entire series of coefficients $b^l_{1,1}$ through a calculation in the worldvolume field theory. Thus, at least in the near infrared, the field theory lagrangian (1.5) appears to provide a holographic description of the physics on the supergravity side, where the geometry is slightly deformed from pure AdS.

We stress that we use the pure AdS/CFT correspondence as a tool to perform the calculations in strongly coupled $\mathcal{N} = 4$ SYM theory. However, since all the required correlators are believed to obey non-renormalization theorems, the computations could equivalently be performed in the free field theory limit by a direct perturbative calculation. (We elaborate on this point in section 4.)

In section 2, we review the supergravity derivation of the coefficients $b^l_{1,1}$ to obtain the explicit expression (1.6). In section 3, we calculate the leading corrections to the two point function in the worldvolume theory defined by (1.5), and show that the results precisely match those from supergravity. In section 4, we offer some concluding remarks.

2. Supergravity calculation of the corrections to dilaton absorption

The coefficients $b^l_{1,1}$ have been implicitly determined in [11] in terms of associated Mathieu functions.[5] Here we sketch an alternative derivation, which is a straightforward extension of the methods of [17].

[5] They may be determined explicitly from [11, eq. (22)] using the first correction to χ which may be found as [15, eq. (A.18)].

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The wave equation for the l-th partial wave of a minimally coupled scalar is

$$\left[\rho^{-5}\frac{d}{d\rho}\rho^5\frac{d}{d\rho}+1-\frac{l(l+4)}{\rho^2}+\frac{(\omega R)^4}{\rho^4}\right]\phi(\rho)=0\,. \tag{2.1}$$

Here $\rho=\omega r$, where r is the standard radial coordinate that enters the harmonic function in the D3-brane metric, $H(r)=1+R^4/r^4$. The wave equation is self-dual under the change of variables $y=(\omega R)^2/\rho$, $\phi=y^4\psi$,

$$\left[y^{-5}\frac{d}{dy}y^5\frac{d}{dy}+1-\frac{l(l+4)}{y^2}+\frac{(\omega R)^4}{y^4}\right]\psi(y)=0\,. \tag{2.2}$$

Following [17], we can find solutions perturbatively in $(\omega R)^4$, both in the "inner" region **I** ($\rho\ll 1$) and in the "outer" region **III** ($\rho\gg(\omega R)^2$). To order $(\omega R)^4$,

$$\begin{aligned}
\phi^{\mathbf{I}} &= y^2H^{(1)}_{2+l}(y)+(\omega R)^4\frac{\pi y^2}{2}\int^y\frac{dx}{x^3}H^{(1)}_{2+l}(x)\Big(J_{2+l}(x)N_{2+l}(y)-N_{2+l}(x)J_{2+l}(y)\Big)\,,\\
\frac{\phi^{\mathbf{III}}}{A} &= \frac{J_{2+l}(\rho)}{\rho^2}-(\omega R)^4\frac{\pi}{2\rho^2}\left[\int^\rho\frac{d\sigma}{\sigma^3}J_{2+l}(\sigma)^2N_{2+l}(\rho)-\right.\\
&\qquad\left.-\int^\rho\frac{d\sigma}{\sigma^3}J_{2+l}(\sigma)N_{2+l}(\sigma)J_{2+l}(\rho)\right]\,.
\end{aligned} \tag{2.3}$$

In the inner region, the solution has been chosen to satisfy the boundary condition of purely ingoing flux at the horizon. In the outer region, the relative ratio of the two independent solutions of the second order wave equation has been fixed by the condition that in the transition region $\phi^{\mathbf{III}}$ can be matched to $\phi^{\mathbf{I}}$ by a choice of the overall factor A. The coefficient A is then be determined by requiring exact matching up to order $(\omega R)^4\log(\omega R)$,

$$A=\frac{-i\,4^{2+l}(l+1)!^2\,(l+2)}{\pi(\omega R)^{2l}}\left[1+\frac{(\omega R)^4\log(\omega R)}{2(l+1)(l+2)(l+3)}+O\left((\omega R)^4\right)\right]. \tag{2.4}$$

To extract the leading correction to the low-energy absorption cross section, we simply recall that $\sigma\sim 1/|A|^2$, so that

$$\sigma^l=\sigma^l_0\left(1-\frac{(\omega R)^4\log(\omega R)}{(l+1)(l+2)(l+3)}+O\left((\omega R)^4\right)\right). \tag{2.5}$$

Comparing with (1.2), we see that $b^l_{1,1}$ are given by (1.6).

3. Computation in perturbed SYM theory

In general, the absorption cross section for a canonically normalized field ϕ of frequency ω coupled to the brane by an interaction

$$\int d^4x\ \phi(x,0)\mathcal{O}(x) \tag{3.1}$$

is given by

$$\sigma(\omega) = \frac{1}{2i\omega} \text{ Disc } \Pi(p)|^{-p^2=\omega^2+i\epsilon}_{-p^2=\omega^2-i\epsilon}\,, \tag{3.2}$$

where $\Pi(p)$ is the momentum space two-point function,

$$\Pi(p) = \int d^4x \; e^{ip\cdot x} \, \langle \mathcal{O}(x)\mathcal{O}(0)\rangle \tag{3.3}$$

computed in the worldvolume theory.

In our case, ϕ is the l-th partial wave of the dilaton field and we assume that the (strongly coupled) worldvolume theory is described by an action[6]

$$S = \int d^4x \left(\mathcal{L}_{\text{SYM}} + (\alpha')^2 \mathcal{O}_8\right), \tag{3.4}$$

where we have absorbed the coefficient c into the definition of $\mathcal{O}_8$ such that $\mathcal{O}_8 = c\,\mathcal{O}_8^{\text{BI}}$. The two point function of the operator coupling to the l-th partial wave of the dilaton in this theory is given by

$$\begin{aligned}\left\langle \mathcal{O}^l_\phi(x)\mathcal{O}^l_\phi(0)\right\rangle &= \left\langle \mathcal{O}^l_\phi(x)\; e^{-\int d^4y (\alpha')^2 \mathcal{O}_8(y)} \; \mathcal{O}^l_\phi(0)\right\rangle_{\text{SYM}} \\ &= \left\langle \mathcal{O}^l_\phi(x)\mathcal{O}^l_\phi(0)\right\rangle_{\text{SYM}} - (\alpha')^2 \int d^4y \left\langle \mathcal{O}^l_\phi(x)\mathcal{O}_8(y)\mathcal{O}^l_\phi(0)\right\rangle_{\text{SYM}} + \cdots .\end{aligned}$$

The functional forms of these two and three point functions in $\mathcal{N}=4$ SYM theory are determined completely by conformal invariance, so we may write

$$\left\langle \mathcal{O}^l_\phi(x)\mathcal{O}^l_\phi(0)\right\rangle_{\text{SYM}} = \frac{t_l}{|x|^{2l+8}} \tag{3.5}$$

and

$$\left\langle \mathcal{O}^l_\phi(x)\mathcal{O}_8(y)\mathcal{O}^l_\phi(0)\right\rangle_{\text{SYM}} = \frac{r_l}{|x|^{2l}|y|^8|x-y|^8}\,. \tag{3.6}$$

Using (3.2), the cross section for the l-th partial wave of the dilaton field is therefore given by

$$\sigma = \frac{1}{2i\omega} \text{ Disc} \int d^4x \; e^{ip\cdot x} \left\{ \frac{t_l}{|x|^{2l+8}} - (\alpha')^2 \int d^4y \frac{r_l}{|x|^{2l}|y|^8|x-y|^8} \right\} + \cdots . \tag{3.7}$$

The Fourier transforms and discontinuities are evaluated in the appendix. We find

$$\begin{aligned}\sigma^l &= \frac{\pi^3 t_l}{2^{2l+4}(l+2)!(l+3)!}\omega^{2l+3} + \frac{5\pi^5 r_l (\alpha')^2}{4^{l+2}(l+4)!(l+5)!}\omega^{2l+7}\log\left(\frac{\omega}{\Lambda}\right) + \cdots \\ &= \sigma^l_0 \left\{ 1 + \frac{5\pi^2(\alpha')^2}{(l+3)(l+4)^2(l+5)} \frac{r_l}{t_l}\omega^4 \log\left(\frac{\omega}{\Lambda}\right) + \cdots \right\}.\end{aligned} \tag{3.8}$$

[6]We note here that higher dimension operators, if present in the complete lagrangian, would not contribute to the leading correction that we calculate.

Comparing this with the formula (1.2) for the supergravity result, we see that[7]

$$\left(b^l_{1,1}R^4\right)_{wv} = \frac{5\pi^2(\alpha')^2}{(l+3)(l+4)^2(l+5)}\frac{r_l}{t_l}\,. \tag{3.9}$$

Thus, to evaluate $b^l_{1,1}$ we must evaluate the coefficient of the three point function (3.6). We may take an arbitrary normalization for $\mathcal{O}_\phi$ since we divide by its two point function in the result (3.9), however the result does depend on the normalization of $\mathcal{O}_8$ in the action (3.4). On the other hand, the quantities

$$\frac{b^l_{1,1}}{b^0_{1,1}} = \frac{240}{(l+3)(l+4)^2(l+5)}\frac{r_l t_0}{r_0 t_l} \tag{3.10}$$

are completely independent of the normalizations used for the operators, so these may be compared directly with the supergravity result without worrying about the normalization of $\mathcal{O}_8$.

To perform this comparison, it only remains to determine r_l/t_l by computing the correlators (3.5) and (3.6) in any convenient normalization. Fortunately, these exact correlation functions were calculated in [8] via the AdS/CFT correspondence, using the fact that $\mathcal{O}_l$ corresponds to the l-th Kaluza-Klein mode of the dilaton field, while $\mathcal{O}_8$ corresponds to the dilation mode of the five-sphere. In terms of the normalization independent quantity $(r_l t_0)/(r_0 t_l)$, the result is

$$\frac{r_l t_0}{r_0 t_l} = \frac{(l+4)^2(l+5)}{40(l+1)(l+2)}\,. \tag{3.11}$$

Using (3.10), the worldvolume field theory prediction for $b^l_{1,1}/b^0_{1,1}$ is therefore

$$\left(\frac{b^l_{1,1}}{b^0_{1,1}}\right)_{wv} = \frac{6}{(l+1)(l+2)(l+3)}\,, \tag{3.12}$$

which precisely agrees with the supergravity result (1.6).

Another way to state this result is that by choosing the normalization of $\mathcal{O}_8$ in the strong coupling lagrangian such that leading correction to the s-wave absorption cross section matches the supergravity result, the leading corrections for all other partial waves are also correctly reproduced.

Actually, using the supergravity result for the s-wave correction, $b^0_{1,1} = -1/6$, we are able to to give the normalization of $\mathcal{O}_8$ in the strong coupling lagrangian explicitly. We note that eq. (3.9) implies

$$\frac{\pi^2}{48}(\alpha')^2\frac{r_0}{t_0} = b^0_{1,1}R^4 = -\frac{1}{6}(4\pi g N\left(\alpha'\right)^2)\,. \tag{3.13}$$

[7]Note that we use euclidean conventions in this section, so r_l is negative.

On the other hand, Liu and Tseytlin [18] have computed

$$\frac{r_0}{t_0 k} = \frac{4}{N\sqrt{105}}\,, \tag{3.14}$$

where k is defined by

$$\langle \mathcal{O}_8(x)\mathcal{O}_8(0)\rangle = \frac{k^2}{|x|^{16}}\,. \tag{3.15}$$

Combining these, we conclude that the normalization of $\mathcal{O}_8$ in the strong coupling lagrangian (3.4) is specified by (3.15) with

$$k^2 = \frac{6720 N^2 (Ng)^2}{\pi^2}\,. \tag{3.16}$$

4. Remarks

We have demonstrated that the leading corrections to the low-energy absorption cross sections for all partial waves of the dilaton field may be reproduced by a field theory calculation with the lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SYM} + (\alpha')^2 \mathcal{O}_8\,, \tag{4.1}$$

where $\mathcal{O}_8$ is the unique Lorentz and SO(6) scalar dimension eight operator in a short multiplet of the $\mathcal{N}=4$ superconformal algebra, normalized so that

$$\langle \mathcal{O}_8(x)\mathcal{O}_8(0)\rangle = \frac{6720 N^2 (Ng)^2}{\pi^2}\frac{1}{|x|^{16}}\,. \tag{4.2}$$

It is interesting to note that all correlators used in our computations (two and three point functions of chiral operators of $\mathcal{N}=4$ SYM) are believed to obey non-renormalization theorems, so that in principle, the calculations could all have been performed in the free field theory approximation without relying on the AdS/CFT correspondence. In practice, the computation of the required three point functions for all but the s-wave case would require knowledge of the scalar and fermion terms in $\mathcal{O}_8$, and these have not yet been determined explicitly.

Several methods have been proposed to obtain the precise field theory expressions of operators dual to supergravity modes.[8] We would like to point out that in the specific case of $\mathcal{O}_8$, much of the relevant information is already available from studies of supersymmetry in the low-energy open string theory effective action [22]–[25]. Metsaev and Rahmanov [23] considered ten-dimensional U(1) SYM theory perturbed

[8]We mention: correspondence with M(atrix) theory results [19, 6]; use of the representation theory of the $\mathcal{N}=4$ superconformal algebra and superspace techniques [20]; expansion of the DBI action in AdS background [21]; expansion of the flat space DBI action in terms of the modes determined by the "matching" procedure in the supergravity absorption calculation [19].

by the most general combination of irrelevant operators up to dimension eight, and showed that supersymmetry restricts the form of the action to be

$$S_{\rm 10d} = \int d^{10}x \; \mathcal{L}_{\rm SYM} + a\,\alpha'^2\,\mathcal{O}_8^{\rm 10d} + O\left(\alpha'^3\right), \tag{4.3}$$

where $\mathcal{O}_8^{\rm 10d}$ is completely determined by supersymmetry and explicitly given in terms of ten-dimensional fields, including fermions.[9] For a specific value of a this action correctly reproduces the string amplitudes to order $O(\alpha'^2)$. The operator $\mathcal{O}_8^{\rm BI}$ that appears in the four-dimensional abelian Born-Infeld action is (by definition) the dimensional reduction of $\mathcal{O}_8^{\rm 10d}$. The non-abelian SU(N) color structure is then introduced by the symmetrized trace prescription, which is known to yield the correct string amplitudes to this order in α' [22]. However there is an important caveat: this method (as any other method that uses an integrated action) determines $\mathcal{O}_8$ only up to total derivative terms. This is immaterial for the contribution of the irrelevant perturbation to the dilaton two point functions considered in this paper, but is important if one is interested in the value of the two point function $\langle\mathcal{O}_8(x)\mathcal{O}_8(y)\rangle$. The total derivative terms can be fixed by requiring that $\mathcal{O}_8$ is a primary field of the four-dimensional bosonic conformal group, i.e. is annihilated by the special conformal generators K_μ. Only such an operator would lead to the usual conformal space-time dependence of two and three point functions. It is worth pointing out that even the standard expression $F^4 - 1/4(F^2)^2$ for the bosonic part of $\mathcal{O}_8$ (here F denotes the ten-dimensional field strength) does not have this property, and should be corrected by adding the appropriate total derivatives required by conformal invariance.

For the s-wave case, the required three point correlation function may be calculated perturbatively using only the known gauge field parts of $\mathcal{O}_8$, however one must independently determine the correct normalization for $\mathcal{O}_8$ in the lagrangian to make a non-trivial prediction. In fact, such a calculation was attempted in [17] assuming that the correct normalization for $\mathcal{O}_8$ could be taken from the weak coupling Born-Infeld lagrangian.[10] Their result for $b^0_{1,1}$ from the field theory calculation was 3/4 of the supergravity result, however the discrepancy was partly due to an incorrect treatment of the non-abelian color structure of $\mathcal{O}_8$. Using the correct prescription, it turns out that the field theory result is exactly 1/2 of the supergravity result. Since the correlators themselves are not renormalized, the most likely explanation for this discrepancy is that the normalization of $\mathcal{O}_8$ in the weak coupling lagrangian is exactly 1/2 of its normalization in the strong coupling lagrangian. If all this is

[9]The action (4.3) is invariant under a modified supersymmetry $\delta = \delta_0 + \alpha'^2\delta_1 + \cdots$, where δ_0 is the usual supersymmetry of the unperturbed SYM theory. A simple argument shows then that $\delta_0\,\mathcal{O}_8^{\rm 10d} = 0$ (up to total derivatives), when the SYM equations of motions are imposed. This identifies $\mathcal{O}_8^{\rm 10d}$ as the top member of a supermultiplet.

[10]A similar attempt, but using the DBI action in AdS background, was undertaken in [18].

correct, we are able to make a prediction that

$$\left\langle \mathcal{O}_8^{\mathrm{BI}}(x)\mathcal{O}_8^{\mathrm{BI}}(0)\right\rangle = \frac{1680N^2(Ng)^2}{\pi^2}\frac{1}{|x|^{16}}\,, \tag{4.4}$$

where $\mathcal{O}_8^{\mathrm{BI}}$ is the dimension 8 term in the Born Infeld lagrangian, given in (1.4), with the total derivative ambiguity in $\mathcal{O}_8^{\mathrm{BI}}$ fixed by the requirement of conformal invariance, as explained above.

A natural extension of the work presented here would be to try to reproduce further correction terms in (1.2) using the lagrangian (4.1). It seems plausible that the higher order correction terms of the form $b^l_{n,n}$ are independent of regularization, so that in principle they could be derived in a worldvolume calculation and directly compared with the supergravity results. If a field theory calculation using only the action (4.1) matched with the supergravity prediction (even for $b^0_{2,2}$), it would be even stronger evidence for the Gubser-Hashimoto proposal, since generically, one would expect possible higher dimension operators in the action to contribute here.

On the other hand, as mentioned in the appendix, the terms $b_{1,0}$ come from subleading, regulation dependent terms in the discontinuities of the correlators. We do not have an unambiguous way to evaluate them in order to compare with the supergravity result. The problem of the proper UV definition of the action (4.1) remains a fundamental one. It would be very interesting to see whether by demanding that the $b^l_{1,0}$ terms (and other regularization dependent terms) are correctly reproduced, one is led to a natural renormalization scheme for (4.1) that would make it well defined as a Wilsonian action.

Acknowledgments

We would like to thank Dan Freedman, Steve Gubser, Ken Intriligator, Igor Klebanov, Kostas Skenderis, and Wati Taylor for helpful discussions. The work of L.R. is supported in part by Princeton University "Dicke Fellowship" and by NSF grant 9802484. The work of M.V.R is supported in part by the Stanford Institute for Theoretical Physics and by NSF grant 9870115 .

A. Evaluation of Fourier transforms and discontinuities.

In this appendix, we determine the discontinuity across the positive real axis in the functions

$$f_l(p^2) = \int d^4x\; e^{ip\cdot x}\frac{1}{|x|^{2l+8}} \tag{A.1}$$

and

$$g_l(p^2) = \int d^4x\; e^{ip\cdot x}\int d^4y\frac{1}{|x|^{2l}|x-y|^8|y|^8}\,. \tag{A.2}$$

To do this, we make use of [11, eq. (41)],

$$\mathrm{Disc}\int d^4x \frac{e^{ip\cdot x}(\mu x)^{2a}}{x^{2n}} = \left(\frac{\omega^2}{4}\right)^{n-2}\left(\frac{4\mu^2}{\omega^2}\right)^{a}\frac{2\pi^3 i}{\Gamma(n-a)\Gamma(n-a-1)}\,. \tag{A.3}$$

Using this relation for $a=0$, we find immediately that

$$\mathrm{Disc}\ f_l(p^2)|_{p^2=\omega^2-i\epsilon}^{p^2=\omega^2+i\epsilon} = 2\pi i\frac{\pi^2\omega^{2l+4}}{4^{l+2}(l+2)!(l+3)!}\,. \tag{A.4}$$

To evaluate $g_l(p^2)$, we first perform the integral over y. This requires regularization, which we implement with the cutoff $|y|, |y-x| > 1/\Lambda$. The resulting integral has leading terms with various positive powers of Λ. As standard in field theory, these divergent contributions could be reabsorbed with local counterterms. This requires a renormalization prescription, and we do not have an a priori natural choice. However, the correction to the cross section that we are interested in (proportional to $\omega^4\log(\omega)$) arises from the term logarithmic in x^2, whose coefficient is independent of the cutoff. Explicit computation gives

$$\frac{40\pi^2}{x^{2l+12}}\log(x^2\Lambda^2)\,. \tag{A.5}$$

The discontinuity in the Fourier transform of this function may be read off from the order a term in the expansion of eq. (A.3), and we find

$$\mathrm{Disc}\ g_l(p^2)|_{p^2=\omega^2-i\epsilon}^{p^2=\omega^2+i\epsilon} = -2\pi i\frac{5\pi^4}{2^{2l+5}(l+4)!(l+5)!}\omega^{2l+8}\log\left(\frac{\omega^2}{\Lambda^2}\right)+\cdots\,. \tag{A.6}$$

The dots denote a single additional term proportional to p^{2l+8} without a logarithm. This term is responsible for the $b^l_{1,0}$ term in the cross section, however it is dependent on the renormalization prescription. In contrast, the coefficient of the leading term in the discontinuity is universal. A way to understand the universality of the logarithmic coefficient is its relation with the conformal anomaly of the field theory in the presence of external sources for composite operators, see e.g. [26].[11]

References

[1] I.R. Klebanov, *World-volume approach to absorption by non-dilatonic branes*, *Nucl. Phys.* **B 496** (1997) 231 [hep-th/9702076].

[2] S.S. Gubser and I.R. Klebanov, *Absorption by branes and schwinger terms in the world volume theory*, *Phys. Lett.* **B 413** (1997) 41 [hep-th/9708005].

[3] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, *String theory and classical absorption by three-branes*, *Nucl. Phys.* **B 499** (1997) 217 [hep-th/9703040].

[11] We thank K. Skenderis for discussions on this point.

[4] S.D. Mathur and A. Matusis, *Absorption of partial waves by three-branes*, hep-th/9805064.

[5] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[6] I. Klebanov, I.W. Taylor and M.V. Raamsdonk, *Absorption of dilaton partial waves by D3-branes*, *Nucl. Phys.* **B 560** (1999) 207 [hep-th/9905174].

[7] S. Lee, S. Minwalla, M. Rangamani and N. Seiberg, *Three-point functions of chiral operators in $D = 4$, $N = 4$ SYM at large-N*, *Adv. Theor. Math. Phys.* **2** (1998) 697 [hep-th/9806074];
E. D'Hoker, D.Z. Freedman and W. Skiba, *Field theory tests for correlators in the AdS/CFT correspondence*, *Phys. Rev.* **D 59** (1999) 045008 [hep-th/9807098];
W. Skiba, *Correlators of short multi-trace operators in $N = 4$ supersymmetric Yang-Mills*, *Phys. Rev.* **D 60** (1999) 105038 [hep-th/9907088];
F. Gonzalez-Rey, B. Kulik and I.Y. Park, *Non-renormalization of two point and three point correlators of $N = 4$ SYM in $N = 1$ superspace*, *Phys. Lett.* **B 455** (1999) 164 [hep-th/9903094];
S. Penati, A. Santambrogio and D. Zanon, *Two-point functions of chiral operators in $N = 4$ SYM at order g^4*, *J. High Energy Phys.* **12** (1999) 006 [hep-th/9910197];
K. Intriligator, *Bonus symmetries of $N = 4$ super Yang-Mills correlation functions via AdS duality*, *Nucl. Phys.* **B 551** (1999) 575 [hep-th/9811047];
B. Eden, P.S. Howe and P.C. West, *Nilpotent invariants in $N = 4$ sym*, *Phys. Lett.* **B 463** (1999) 19 [hep-th/9905085];
P.S. Howe, C. Schubert, E. Sokatchev and P.C. West, *Explicit construction of nilpotent covariants in $N = 4$ sym*, *Nucl. Phys.* **B 571** (2000) 71 [hep-th/9910011].

[8] E. D'Hoker, D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Extremal correlators in the AdS/CFT correspondence*, in *The many faces of the superworld*, M.A. Shifman ed., World Scientific, Singapore 2000 [hep-th/9908160].

[9] M. Bianchi and S. Kovacs, *Non-renormalization of extremal correlators in $N = 4$ SYM theory*, *Phys. Lett.* **B 468** (1999) 102 [hep-th/9910016];
B. Eden, P.S. Howe, C. Schubert, E. Sokatchev and P.C. West, *Extremal correlators in four-dimensional SCFT*, *Phys. Lett.* **B 472** (2000) 323 [hep-th/9910150];
J. Erdmenger and M. Perez-Victoria, *Non-renormalization of next-to-extremal correlators in $N = 4$ SYM and the AdS/CFT correspondence*, *Phys. Rev.* **D 62** (2000) 045008 [hep-th/9912250];
E. D'Hoker, J. Erdmenger, D.Z. Freedman and M. Perez-Victoria, *Near-extremal correlators and vanishing supergravity couplings in AdS/CFT*, *Nucl. Phys.* **B 589** (2000) 3 [hep-th/0003218].

[10] B. Eden, A.C. Petkou, C. Schubert and E. Sokatchev, *Partial non-renormalisation of the stress-tensor four-point function in $N = 4$ SYM and AdS/CFT*, hep-th/0009106; G. Arutyunov, S. Frolov and A. Petkou, *Perturbative and instanton corrections to the OPE of CPOs in $N = 4$ SYM_4*, hep-th/0010137.

[11] S.S. Gubser and A. Hashimoto, *Exact absorption probabilities for the D3-brane*, *Comm. Math. Phys.* **203** (1999) 325 [hep-th/9805140].

[12] M. Cvetič, H. Lu and J.F. Vazquez-Poritz, *Massive-scalar absorption by extremal p-branes*, *Phys. Lett.* **B 462** (1999) 62 [hep-th/9904135]; *Absorption by extremal D3-branes*, hep-th/0002128;
R. Manvelian, H.J. W. Muller-Kirsten, J.Q. Liang and Y. bo Zhang, *Absorption cross section of scalar field in supergravity background*, *Nucl. Phys.* **B 579** (2000) 177 [hep-th/0001179].

[13] M.S. Costa, *Absorption by double-centered d3-branes and the Coulomb branch of $N = 4$ SYM theory*, *J. High Energy Phys.* **05** (2000) 041 [hep-th/9912073]; *A test of the AdS/CFT duality on the Coulomb branch*, *Phys. Lett.* **B 482** (2000) 287 [hep-th/0003289].

[14] K. Intriligator, *Maximally supersymmetric RG flows and AdS duality*, *Nucl. Phys.* **B 580** (2000) 99 [hep-th/9909082].

[15] U.H. Danielsson, A. Guijosa, M. Kruczenski and B. Sundborg, *D3-brane holography*, *J. High Energy Phys.* **05** (2000) 028 [hep-th/0004187].

[16] A. Hashimoto, *Holographic description of D3-branes in flat space*, *Phys. Rev.* **D 60** (1999) 127902 [hep-th/9903227].

[17] S.S. Gubser, A. Hashimoto, I.R. Klebanov and M. Krasnitz, *Scalar absorption and the breaking of the world volume conformal invariance*, *Nucl. Phys.* **B 526** (1998) 393 [hep-th/9803023].

[18] H. Liu and A.A. Tseytlin, *Dilaton-fixed scalar correlators and $AdS_5 \times S^5$-SYM correspondence*, *J. High Energy Phys.* **10** (1999) 003 [hep-th/9906151].

[19] I.W. Taylor and M.V. Raamsdonk, *Multiple Dp-branes in weak background fields*, *Nucl. Phys.* **B 573** (2000) 703 [hep-th/9910052].

[20] S. Ferrara, C. Fronsdal and A. Zaffaroni, *On $N = 8$ supergravity on AdS_5 and $N = 4$ superconformal Yang-Mills theory*, *Nucl. Phys.* **B 532** (1998) 153 [hep-th/9802203];
L. Andrianopoli and S. Ferrara, *K-K excitations on $AdS_5 \times S^5$ as $N = 4$ 'primary' superfields*, *Phys. Lett.* **B 430** (1998) 248 [hep-th/9803171];
S. Ferrara, M.A. Lledo and A. Zaffaroni, *Born-Infeld corrections to D3 brane action in $AdS_5 \times S^5$ and $N = 4$, $D = 4$ primary superfields*, *Phys. Rev.* **D 58** (1998) 105029 [hep-th/9805082];
M. Gunaydin, D. Minic and M. Zagermann, *4d doubleton conformal theories, CPT and IIB string on $AdS_5 \times S^5$*, *Nucl. Phys.* **B 534** (1998) 96 [hep-th/9806042].

[21] S.R. Das and S.P. Trivedi, *Three brane action and the correspondence between $N = 4$ Yang-Mills theory and anti de Sitter space*, *Phys. Lett.* **B 445** (1998) 142 [hep-th/9804149];
S.P. de Alwis, *Supergravity, the DBI action and black hole physics*, *Phys. Lett.* **B 435** (1998) 31 [hep-th/9804019];
I.Y. Park, A. Sadrzadeh and T.A. Tuan, *Super Yang-Mills operators from the D3-brane action in a curved background*, hep-th/0010116.

[22] A.A. Tseytlin, *Born-Infeld action, supersymmetry and string theory*, hep-th/9908105.

[23] R.R. Metsaev and M.A. Rakhmanov, *Fermionic terms in the open superstring effective action*, *Phys. Lett.* **B 193** (1987) 202.

[24] E. Bergshoeff, M. Rakowski and E. Sezgin, *Higher derivative super Yang-Mills theories*, *Phys. Lett.* **B 185** (1987) 371.

[25] R.R. Metsaev, M.A. Rakhmanov and A.A. Tseytlin, *The Born-Infeld action as the effective action in the open superstring theory*, *Phys. Lett.* **B 193** (1987) 207.

[26] A. Petkou and K. Skenderis, *A non-renormalization theorem for conformal anomalies*, *Nucl. Phys.* **B 561** (1999) 100 [hep-th/9906030].